

## C\*- Algebras of Integral Operators



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### Objectives

Given the C\*-algebra  $\mathbf{B} = \text{alg}(\text{PSO}, \mathbf{S}, \text{UG})$ , of Cauchy singular integral operators with shifts acting on the unit circle, we want to investigate the **invertibility** and the **Fredholm property** of its elements.

### Methodology

The C\*-algebra  $\mathbf{B} = \text{alg}(\text{PSO}, \mathbf{S}, \text{UG})$  is an algebra associated with a C\*-dynamical system and making convenient decompositions of the **maximal ideal space** of a central C\*-subalgebra  $\mathbf{Z}$  of  $\mathbf{B}$ , it is possible associate to each operator  $\mathbf{b}$  in  $\mathbf{B}$  a family of new operators  $\mathbf{b}_1, \mathbf{b}_2, \dots, \mathbf{b}_k$  (related with the fixed points of shifts  $\mathbf{g}$ ) belonging to simpler C\*-algebras, such that

$\mathbf{B}$  is Fredholm (invertible) **iff**  $\mathbf{b}_1, \mathbf{b}_2, \dots, \mathbf{b}_k$  are all Fredholm (invertible).

### Expected Results

**Find new methods** to construct **Fredholm** (invertible) **Symbols** for C\*-algebras of type  $\mathbf{B}$ , that is, to construct maps  $\Phi$  that associate to each operator  $\mathbf{b}$  in  $\mathbf{B}$  a family of square matrices  $\mathbf{b}_1, \mathbf{b}_2, \dots, \mathbf{b}_k$ ,

$$\Phi : B \rightarrow M_{n_1}(\mathbb{C}) \oplus M_{n_2}(\mathbb{C}) \oplus \dots \oplus M_{n_k}(\mathbb{C})$$

$$b \mapsto (b_1, b_2, \dots, b_k)$$

such that  $\mathbf{b}$  is Fredholm (invertible) **iff** all the **determinants** of matrices  $\mathbf{b}_1, \mathbf{b}_2, \dots, \mathbf{b}_k$  are nonzero.

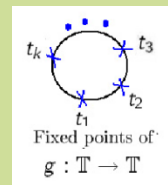


Figure 1: Fixed points

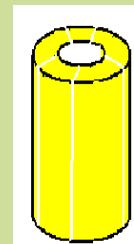


Figure 2: Maximal ideal space

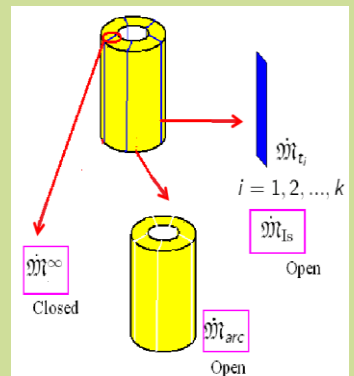


Figure 3: Decomposition which leads to new operators