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Pseudodifferential Operators

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Objectives

The aim of this ongoing project is to establish the boundedness and Fredholmness of pseudodifferential operators with rough symbols on Banach function spaces in the sense of Luxemburg. This class of Banach spaces includes, for instance, classical Lebesgue and Orlicz spaces, as well as, generalized Lebesgue spaces with variable exponents. Note that generalized Lebesgue spaces with variable exponents are important in Fluid Dynamics because it is a natural setting to study non-Newtonian fluids, fluids whose local properties depend on a chosen direction.

Methodology

We are going to look at the boundedness problem from the point of veiw of Harmonic Analysis, studying pseudofferential operators as integral operators with kernels satisfying some regularity and cancellation properties. Pointwise estimates of the Fefferman-Stein sharp maximal function of pseudodifferential operators via the the Hardy-Littlewood maximal functions are important tools in our study. Further, we plan to embark on the study of Fredholmness (=invertibility modulo the compact operators). Here analogs of the Krasnoselskii compactness interpolation theorem will be important.

Keywords:
Pseudodifferential operator;
Rough symbol;
Fefferman-Stein maximal function;
Hardy-Littlewood maximal function;
Banach function space;
Generalized Lebesgue space with variable exponent;
Fredholmness

Expected Results

We expect to get a pointwise estimate for a the Fefferman-Stein maximal function of a pseudodifferential operator (PDO) of a nice (smooth) function via its Hardy-Littlewood maximal function and a seminorm of a symbol of PDO. This will lead to corresponding estimates for the norms in Banach function spaces. Further we plan to get na effectively veifiable necessary and sufficint condition for the Fredholmness (=invertibility modulo the compact operators) of PDO with a rough symbol in a certain sufficiently wide class.