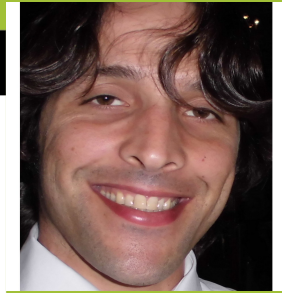


Algebraic and Differential-Geometric Methods for Topological Quantum Field Theories

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Objectives

- 1 Formulate the categorifications of BF-Theory and of Chern-Simons theory and address the applications for defining invariants of knots and manifolds.
- 1 Understand Gray-categories and how they arise in Geometry and Topology.
- 1 Understand the categorification of the Kontsevich algebra of Jacobi diagrams. Understand the categorification of the associated 4-terms relation, giving the definition of an infinitesimal 2-R-matrix.

Methodology

- 1 Investigate the path-integrals associated to the categorifications of BF-Theory and of Chern-Simons Theory in two different ways:
 - 1 - Combinatorially (via spin foam state sum models).
 - 1 - As a perturbation series (via differential-geometric methods).
- 1 In both cases higher categories, and in particular Gray categories, should have an essential role by providing the overall algebraic framework.
- 1 Defining bundles with a Gray-groupoid of structure and investigate the associated bi-categorified BF-Theory.

Expected Results

- 1 Given an embedded 2-sphere in the 4-sphere define a perturbation series for categorified BF-theory, considering Wilson-surface observables. This is to be a topological invariant.
- 1 Define a spin-foam state sum model for the path-integral of categorified BF-theory in a four-manifold.
- 1 Definition of the tri-category of Gray-bundles (with tri-connections) and their 1st, 2nd and 3rd order gauge transformations
- 1 Prove that the categorified 4-terms relation arise from linear braided monoidal 2-categories as a classical limit. Prove that any infinitesimal 2-R-matrix can be quantised (possibly via Drinfeld 2-associators).