

Mathematics Department

## Optimization

CMA – Operational Research Group



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## Objectives

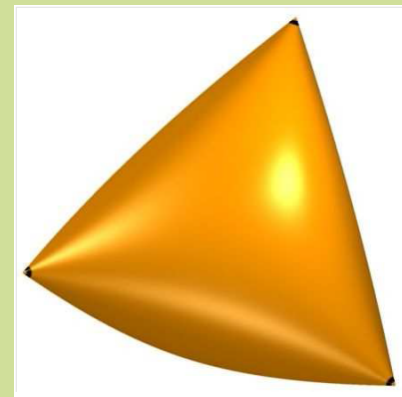
We investigate new connections between the SAT problem and an optimization model (SDP) of it. It is known that if the SDP relaxation of a SAT instance is infeasible, then the SAT instance is unsatisfiable. When the SDP relaxation is infeasible, we can exhibit a proof in the form of a certificate of infeasibility. Until now, this certificate was only used to prove unsatisfiability, but no other information was extracted from it.

The objective is to address the open problem of how to extract the combinatorial information contained in the certificate of an arbitrary unsatisfiable instance.

## Methodology

The certificate of infeasibility is theoretically supported by what is known as Farkas' Lemma, which certifies if a linear system is inconsistent. This certificate can be obtained as the non-negative least squares problem (residual vector of the linear constraints representing clauses).

Each coordinate of the certificate identifies uniquely each clause in the original SAT instance. We note that interior-point software provides this numerical certificate when the SDP instance is infeasible.



## Expected Results

We expect to interpret each entry of the certificate of infeasibility as a measure of "hardness" to satisfy clauses in the propositional formula. The key here is to identify the properties in the certificate that correspond to the underlying minimal unsatisfiable sub-formula. Thus, we will compare and classify, from a theoretical perspective, the weights of each clause and obtain the criteria to identify different MUSes.

We expect to devise an algorithm to extract the information from the numerical certificate.

