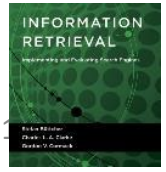
Language Models

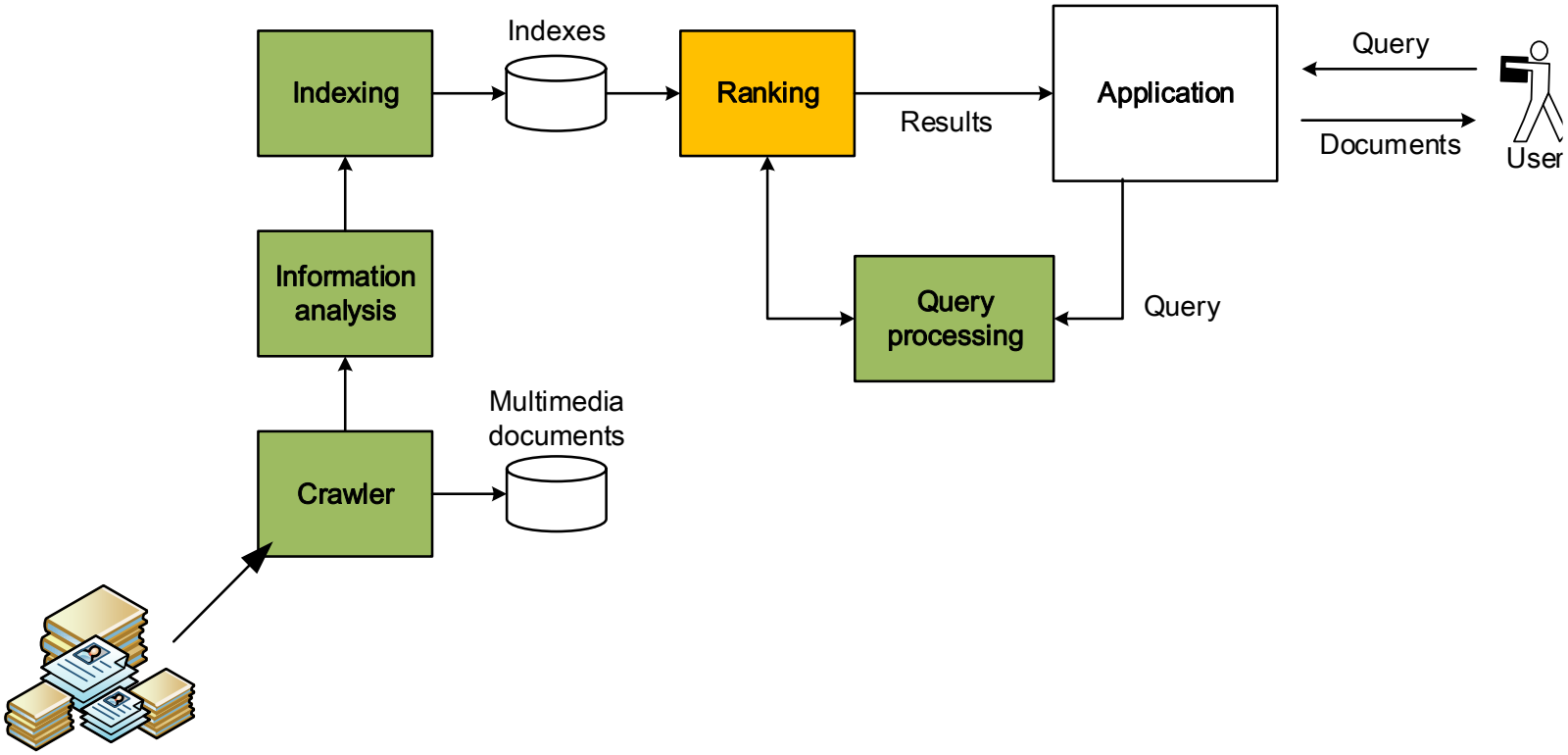
LM Jelinek-Mercer Smoothing and LM Dirichlet Smoothing

Web Search

Slides based on the books:



Overview


Indexing
Indexes
Ranking
Results
Application
Query
Documents
User
Information analysis
Query processing
Query
Multimedia documents
Crawler

What is a language model?

- We can view a finite state automaton as a deterministic language model.

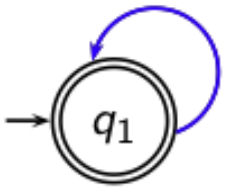
- I wish I wish I wish I wish . . . Cannot generate: “wish I wish” or “I wish I”.

- Our basic model: each document was generated by a different automaton like this except that these automata are probabilistic.

15

A probabilistic language model

- This is a one-state probabilistic finite-state automaton (a unigram LM) and the state emission distribution for its one state q1.
  - STOP is not a word, but a special symbol indicating that the automaton stops.



w	$P(w\|q_1)$	w	$P(w\|q_1)$
STOP	0.2	toad	0.01
the	0.2	said	0.03
a	0.1	likes	0.02
frog	0.01	that	0.04
		...	...

String = “frog said that toad likes frog STOP”

P(string) = 0.01 · 0.03 · 0.04 · 0.01 · 0.02 · 0.01 · 0.2= 0.0000000000048

A language model per document

language model of d_1

w	$P(w\|.)$	w	$P(w\|.)$
STOP	.2	toad	.01
the	.2	said	.03
a	.1	likes	.02
frog	.01	that	.04
		...	...

language model of d_2

w	$P(w\|.)$	w	$P(w\|.)$
STOP	.2	toad	.02
the	.15	said	.03
a	.08	likes	.02
frog	.01	that	.05
		...	...

String = “frog said that toad likes frog STOP”

P(string|Md1) = $0.01 \cdot 0.03 \cdot 0.04 \cdot 0.01 \cdot 0.02 \cdot 0.01 \cdot 0.02 = 0.0000000000048 = 4.8 \cdot 10^{-12}$

P(string|Md2) = $0.01 \cdot 0.03 \cdot 0.05 \cdot 0.02 \cdot 0.02 \cdot 0.01 \cdot 0.02 = 0.0000000000120 = 12 \cdot 10^{-12}$

P(string|Md1) < P(string|Md2)

- Thus, document d2 is “more relevant” to the string “frog said that toad likes frog STOP” than d1 is.

Types of language models

- Unigrams: $p_{uni}(t_1 t_2 t_3 t_4) = p(t_1)p(t_2)p(t_3)p(t_4)$

- Bigrams: $p_{bi}(t_1 t_2 t_3 t_4) = p(t_1)p(t_2|t_1)p(t_3|t_2)p(t_4|t_3)$

- Multinomial distributions over words:

$$p(d) = \frac{l_d!}{f_{t_1,d}! f_{t_2,d}! \ldots f_{t_n,d}!} p(t_1)^{f_{t_1,d}} p(t_2)^{f_{t_2,d}} \ldots p(t_n)^{f_{t_n,d}}$$

Probability Ranking Principle (PRP)

- PRP in action: Rank all documents by $p(r = 1|q, d)$
  - Theorem: Using the PRP is optimal, in that it minimizes the loss (Bayes risk) under 1/0 loss
  - Provable if all probabilities correct, etc. [e.g., Ripley 1996]

$$p(r|q, d) = \frac{p(d, q|r)p(r)}{p(d, q)}$$

- Using odds, we reach a more convenient formulation of ranking :

$$\mathrm{O}(R|q, d) = \frac{p(r = 1|q, d)}{p(r = 0|q, d)}$$

Language models

- In language models, we do a different formulation towards the query posterior given the document as a model.

$$\mathrm{O}(R|q,d) = \frac{p(r=1|q,d)}{p(r=0|q,d)} = \frac{p(d,q|r=1)p(r=1)}{p(d,q|r=0)p(r=0)}$$

$$= \frac{p(q|d,r)p(d|r)p(r)}{p(q|d,\bar{r})p(d|\bar{r})p(\bar{r})} \propto \log \frac{p(q|d,r\,)p(r|d)}{p(q|d,\bar{r})p(\bar{r}|d)}$$

$$= \log p(q|d,r) - \log p(q|d,\bar{r}) + \log \frac{p(r|d)}{p(\bar{r}|d)}$$

Language models

$$\log p(q|d,r) - \log p(q|d,\bar{r}) + \log \frac{p(r|d)}{p(\bar{r}|d)}$$

$$\approx \log p(q|d,r) + \text{logit}\big(p(r|d)\big)$$

- The fist term computes the probability that the query has been generated by the document model
- The second term can measure the quality of the document with respect to other indicators not contained in the query (e.g. PageRank or number of links)

How to compute $p(q|d)$?

- We will make the same conditional independence assumption as for Naive Bayes (we dropped the *r* variable)

$$p(q|M_d) = \prod_{i=0}^{|q|} p(t_i|M_d)$$

  - |q| length of query;
  - t_i the token occurring at position *i* in the query

- This is equivalent to: $p(q|M_d) = \prod_{t \in \{q \cap d\}} p(t|M_d)^{tf_{t,q}}$

  - $tf_{t,q}$ is the term frequency (# occurrences) of t in q
- Multinomial model (omitting constant factor)

Parameter estimation

- The parameters $p(t|M_d)$ are obtained from the document data as the maximum likelihood estimate:

$$p(t|M_d^{ml}) = \frac{f_{t,d}}{|d|}$$

- A single t with $p(t|M_d) = 0$ will make $p(q|M_d) = \prod p(t|M_d)$ zero.

- This can be smoothed with the prior knowledge we have about the collection.

Smoothing

- Key intuition: A non-occurring term is possible (even though it didn't occur), . . .
  - . . . but no more likely than would be expected by chance in the collection.

- The maximum likelihood language model M_C^{ml} based on the term frequencies in the collection as a whole:

$$p\left(t\middle|M_C^{ml}\right) = \frac{l_t}{l_C}$$

  - l_t is the number of times the term shows up in the collection
  - l_C is the number of terms in the whole collection.

- We will use $p\left(t\middle|M_C^{ml}\right)$ to "smooth" $p(t|d)$ away from zero.

LM with Jelineck-Mercer smoothing

- The first approach we can do is to create a mixture model with both distributions:

$$p(q|d, C) = \lambda \cdot p(q|M_d) + (1 - \lambda) \cdot p(q|M_c)$$

- Mixes the probability from the document with the general collection frequency of the word.

- High value of λ: “conjunctive-like” search – tends to retrieve documents containing all query words.
- Low value of λ: more disjunctive, suitable for long queries
- Correctly setting λ is very important for good performance.

Mixture model: Summary

- What we model: *The user has some background knowledge about the collection and has a "document in mind" and generates the query from this document*.

$$p(q|d,C) \approx \prod_{t \in \{q \cap d\}} \left(\lambda \cdot p(t|M_d) + (1-\lambda) \cdot p(t|M_c)\right)$$

- The equation represents the probability that the document that the user had in mind was in fact this one.

LM with Dirichlet smoothing

- We can use the prior knowledge about the mean of each term.

- The mean of the term in the collection should be our starting point when computing the term average on a document:
  - Imagine that we can add a fractional number occurrences to each term frequency.
  - Add $\mu = 1000$ occurrences of terms to a document according to the collection distribution.
  - The frequency of each term t_i would increase $\mu \cdot M_c(t_i)$
  - The length of each document increases by 1000.

- This will change the way we compute the mean of a term on a document.

Dirichlet smoothing

- We end up with the maximum a posteriori estimate of the term average:

$$p\left(t\middle|M_d^{MAP}\right) = \frac{f_{t,d} + \mu \cdot M_c(t)}{|d| + \mu}$$

  - This is equivalent to using a Dirichlet prior with appropriate parameters.

- The ranking function becomes:

$$p(q|d) = \prod_{t \in q} \left(\frac{f_{t,d} + \mu \cdot M_c(t)}{|d| + \mu} \right)^{q_t}$$

Experimental comparison

	TREC45				Gov2			
	1998		1999		2005		2006	
Method	P@10	MAP	P@10	MAP	P@10	MAP	P@10	MAP
Binary	0.256	0.141	0.224	0.148	0.069	0.050	0.106	0.083
2-Poisson	0.402	0.177	0.406	0.207	0.418	0.171	0.538	0.207
BM25	0.424	0.178	0.440	0.205	0.471	0.243	0.534	0.277
LMJM	0.390	0.179	0.432	0.209	0.416	0.211	0.494	0.257
LMD	0.450	0.193	0.428	0.226	0.484	0.244	0.580	0.293
BM25F					0.482	0.242	0.544	0.277
BM25+PRF	0.452	0.239	0.454	0.249	0.567	0.277	0.588	0.314
RRF	0.462	0.215	0.464	0.252	0.543	0.297	0.570	0.352
LR			0.446	0.266			0.588	0.309
RankSVM			0.420	0.234			0.556	0.268

Experimental comparison

- For long queries, the Jelinek-Mercer smoothing performs better than the Dirichlet smoothing.
- For short queries, the Dirichlet smoothing performs better than the Jelinek-Mercer smoothing.

Method	Query	AP	Prec@10	Prec@20
LMJM	Title	0.227	0.323	0.265
LMD	Title	**0.256**	**0.352**	**0.289**
LMJM	Long	**0.280**	**0.388**	**0.315**
LMD	Long	0.279	0.373	0.303

Chengxiang Zhai and John Lafferty. 2004. A study of smoothing methods for language models applied to information retrieval. *ACM Trans. Inf. Syst.* 22, 2 (April 2004), 179-214.

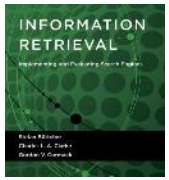
Summary

- Language Models
  - Jelinek-Mercer smoothing
  - Dirichlet smoothing

- Both models need to estimate one single parameter from the whole collection
  - (although there are known values that work well).

- References:

Chapter 12



Sections 9.1, 9.2 and 9.3