Ranking linked data
Web graph, PageRank, Topic-specific PageRank and HITS
Web Search
Overview

Application
Multimedia documents
User
Information analysis
Indexes
Ranking
Query
Documents
Indexing
Query Results
Query processing
Crawler

Indexes
Results
Query
Documents
User
Multimedia documents

Indexing → Indexes
Ranking → Results
Application → Query
Documents → User
Crawler → Multimedia documents
Information analysis → Indexes
Query processing → Query
Ranking linked data

• Links are inserted by humans.

• They are one of the most valuable judgments of a page’s importance.

• A link is inserted to denote an association. The anchor text describes the type of association.
The Web as a directed graph

Assumption 1: A hyperlink between pages denotes author perceived relevance (quality signal)

Assumption 2: The anchor of the hyperlink describes the target page (textual context)
Anchor text

• When indexing a document $D$, include anchor text from links pointing to $D$. 

Armonk, NY-based computer giant **IBM** announced today 

www.ibm.com

Joe’s computer hardware links
Compaq
HP
IBM

**Big Blue** today announced record profits for the quarter
Indexing anchor text

• Can sometimes have unexpected side effects - *e.g.*, *evil empire*.

• Can boost anchor text with weight depending on the authority of the anchor page’s website
  
  • *E.g.*, if we were to assume that content from cnn.com or yahoo.com is authoritative, then trust the anchor text from them
Citation analysis

• Citation frequency

• Co-citation coupling frequency
  • Co-citations with a given author measures “impact”
  • Co-citation analysis [Mcca90]

• Bibliographic coupling frequency
  • Articles that co-cite the same articles are related

• Citation indexing
  • Who is author cited by? [Garf72]

• PageRank preview: Pinsker and Narin ’60s
Incoming and outgoing links

• The popularity of a page is related to the number of incoming links
  • Positively popular
  • Negatively popular

• The popularity of a page is related to the popularity of pages pointing to them
Query-independent ordering

• First generation: using link counts as simple measures of popularity.

• Two basic suggestions:
  • Undirected popularity:
    • Each page gets a score = the number of in-links plus the number of out-links (3+2=5).
  • Directed popularity:
    • Score of a page = number of its in-links (3).
PageRank scoring

• Imagine a browser doing a random walk on web pages:
  • Start at a random page
  • At each step, go out of the current page along one of the links on that page, equiprobably

• “In the steady state” each page has a long-term visit rate - use this as the page’s score.
Not quite enough

• The web is full of dead-ends.
  • Random walk can get stuck in dead-ends.
  • Makes no sense to talk about long-term visit rates.
Teleporting

• At a dead end, jump to a random web page.

• At any non-dead end, with probability 10%, jump to a random web page.
  • With remaining probability (90%), go out on a random link.
  • 10% - a parameter.

• Result of teleporting:
  • Now cannot get stuck locally.
  • There is a long-term rate at which any page is visited.
  • How do we compute this visit rate?
The random surfer

• The PageRank of a page is the probability that a given random “Web surfer” is currently visiting that page.

• This probability is related to the incoming links and to a certain degree of browsing randomness (e.g. reaching a page through a search engine).
Markov chains

• A Markov chain consists of \( n \) states, plus an \( n \times n \) transition probability matrix \( P \).

• At each step, we are in exactly one of the states.

• For \( 1 \leq i,j \leq n \), the matrix entry \( P_{ij} \) tells us the probability of \( j \) being the next state, given we are currently in state \( i \).
Transitions probability matrix

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Ergodic Markov chains

• A Markov chain is **ergodic** if

  • you have a path from any state to any other

  • For any start state, after a finite transient time $T_0$, the probability of being in any state at a fixed time $T > T_0$ is nonzero.

Not ergodic (even/odd).
Ergodic Markov chains

• For any ergodic Markov chain, there is a unique long-term visit rate for each state.
  • Steady-state probability distribution.

• Over a long time-period, we visit each state in proportion to this rate.

• It doesn’t matter where we start.

The PageRank of Web page $i$ corresponds to the probability of being at page $i$ after an infinite random walk across all pages (i.e., the stationary distribution).
PageRank

- The rank of a page is related to the number of incoming links of that page and the rank of the pages linking to it.

\[ PR(A) = (1 - d) + d \cdot \left[ \frac{PR(B)}{OL(B)} + \frac{PR(C)}{OL(C)} \right] \]
PageRank: formalization

• The RandomSurfer model assumes that the pages with more inlinks are visited more often

\[
\begin{align*}
\text{C} & \quad 1 \quad \text{A} \quad 0.5 \\
\quad 0.5 & \quad \text{A} \quad \quad 0.5 \\
& \quad 0.5 \quad \text{B} \\
& \quad 1 \\
\end{align*}
\]

• The rank of a page is computed as:

\[
p_i = (1 - d) + d \sum_{j=1}^{N} \left( \frac{L_{ij}}{c_j} \right) p_j
\]

where \( L_{ij} \) is the link matrix, \( c_j \) is the number of links of page and \( p_j \) is the PageRank of that page
Transitions probability matrix

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Example

• Consider three Web pages:

\[
\begin{array}{c}
\text{C} \quad 1 \quad 0.5 \quad \text{A} \\
0.5 \quad 1 \quad \text{B}
\end{array}
\]

\[
p_i = (1 - d) + d \sum_{j=1}^{N} \left( \frac{L_{ij}}{c_j} \right) p_j
\]

• The transition matrix \( \frac{L_{ij}}{c_j} \) is:

\[
\begin{pmatrix}
0 & 0.5 & 0.5 \\
1 & 0 & 0 \\
1 & 0 & 0
\end{pmatrix}
\]
PageRank: issues and variants

• How realistic is the random surfer model?
  • What if we modeled the back button? [Fagi00]
  • Surfer behavior sharply skewed towards short paths [Hube98]
  • Search engines, bookmarks & directories make jumps non-random.

• Biased Surfer Models
  • Weight edge traversal probabilities based on match with topic/query (non-uniform edge selection)
  • Bias jumps to pages on topic (e.g., based on personal bookmarks & categories of interest)
Topic Specific Pagerank [Have02]

• Conceptually, we use a random surfer who teleports, with ~10% probability, using the following rule:
  • Selects a category (say, one of the 16 top level categories) based on a query & user-specific distribution over the categories
  • Teleport to a page uniformly at random within the chosen category

• Sounds hard to implement: can’t compute PageRank at query time!
Topic Specific PageRank - Implementation

• **offline**: Compute pagerank distributions wrt individual categories
  • Query independent model as before
  • Each page has multiple pagerank scores – one for each category, with teleportation only to that category

• **online**: Distribution of weights over categories computed by query context classification
  • Generate a dynamic pagerank score for each page - weighted sum of category-specific pageranks
Example

• Consider a query on a given set of Web pages with the following graph:

• The query has **90%** probability of being about **Sports**.
• The query has **10%** probability of being about **Health**.
Non-uniform Teleportation

Sports teleportation

Health teleportation
pr = (0.9 \text{PR}_{\text{sports}} + 0.1 \text{PR}_{\text{health}}) \text{ gives you:}
9\% \text{ sports teleportation, } 1\% \text{ health teleportation}
In response to a query, instead of an ordered list of pages each meeting the query, find two sets of inter-related pages:

- **Hub pages** are good lists of links on a subject.
  - e.g., “Bob’s list of cancer-related links.”
- **Authority pages** occur recurrently on good hubs for the subject.

- Best suited for “broad topic” queries rather than for page-finding queries.

- Gets at a broader slice of common opinion.
The hope

Long distance telephone companies

Alice

Bob

AT&T

Sprint

MCI

Hubs

Authorities
High-level scheme

• Extract from the web a base set of pages that could be good hubs or authorities.

• From these, identify a small set of top hub and authority pages;
  • iterative algorithm.
Base set and root set

• Given text query (say *browser*), use a text index to get all pages containing *browser*.
  • Call this the root set of pages.

• Add in any page that either
  • points to a page in the root set, or
  • is pointed to by a page in the root set.

• Call this the base set.
Distilling hubs and authorities

• Compute, for each page $x$ in the base set, a **hub score** $h(x)$ and an **authority score** $a(x)$.

• Initialize: for all $x$, $h(x) \leftarrow 1$; $a(x) \leftarrow 1$;

• Iteratively update all $h(x)$, $a(x)$;

• After iterations
  • output pages with highest $h()$ scores as top hubs
  • highest $a()$ scores as top authorities.
Iterative update

• Repeat the following updates, for all $x$:

\[
a(x) \leftarrow \sum_{y \rightarrow x} h(y)
\]

\[
h(x) \leftarrow \sum_{x \rightarrow y} a(y)
\]
How many iterations?

• Claim: relative values of scores will converge after a few iterations:
  • in fact, suitably scaled, h() and a() scores settle into a steady state!

• We only require the relative orders of the h() and a() scores - not their absolute values.

• In practice, ~5 iterations get you close to stability.
Summary

• Global relevance of an edge in a graph

• Link directions are important

• A few iterations should be enough (you just want to compute the rank of pages, not the absolute value of relevance)

• There are other way to distinguish the type of relevance