

A type checking algorithm for qualified session types

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WWV June 9 2011, Reykjavik

Structured-oriented programming_{SOP}

- Session types are a static analysis technique for service oriented protocols
- Allow for analyzing the message-passing interaction among a service provider and a client
- Introduced for the pi calculus and now embedded also in other paradigms:
 - functional programming
 - object oriented programming
- Idea: allowing typing of communication channels by using polymorphic *sequences* of types as:

```
output Integer . output Boolean . input Boolean . end
```

Qualified session types

- Session types core: input, output and termination

$?T.S$	input
$!T.S$	output
end	termination

- Qualifiers

$\text{lin}?T.S$	linear input
$\text{lin}!T.S$	linear output
$S_1 = \text{un}?T.S_1$	unrestricted recursive input
$S_2 = \text{un}!T.S_2$	unrestricted recursive output
un end	termination

SOP in the pi calculus

- Example: event scheduling (e.g. Doodle)
- Two-steps service protocol for scheduling a meeting (client side)
 1. Ask to create a poll
 - provide a title for the meeting
 - provide a provisional date
 2. Invite participants
- Implementation: send **request** to create poll / **receive** poll channel

$$\overline{\text{poll}}\langle y \rangle . y(\overline{p}) . (\overline{p}\langle \text{Workshop} \rangle . \overline{p}\langle 9\text{June} \rangle . (\overline{z_1}\langle p \rangle \mid \cdots \mid \overline{z_n}\langle p \rangle))$$

- Challenge: concurrent distribution of the poll channel

Session type for the poll

- Poll channel used first in linear mode then in unrestricted mode
 1. Send a title for the poll (**linear mode**)
 2. Send a date for the poll (**linear mode**)
 3. Distribute the poll (**unrestricted mode**)

$$\bar{p}\langle\text{Workshop}\rangle.\bar{p}\langle\text{9June}\rangle.(\bar{z}_1\langle p\rangle \mid \cdots \mid \bar{z}_n\langle p\rangle)$$

- End point session type for channel p is

$$\text{lin !string.lin !date.*un !date}$$

- Recursive type ***un !date** allows for distribution of poll channel

Concurrent session types

- Service: instantiation generates poll

$$Service = !poll(w).(\nu p : T)\bar{w}\langle p \rangle.p(title).p(date).!p(date)$$

- One channel end sent to the **invoker**

$$S_1 = \text{lin} !\text{string}.\text{lin} !\text{date}.*\text{un} !\text{date}$$

- The other channel end used in the **continuation**

$$S_2 = \text{lin} ?\text{string}.\text{lin} ?\text{date}.*\text{un} ?\text{date}$$

- Full type for p describes *concurrent* behavior of two channel ends

$$T = (S_1, S_2)$$

This talk

- We present a type checking algorithm for qualified session types of the form

$$T = (S_1, S_2)$$

- The algorithm can be seen as an implementation of type system \vdash of [G&V@Concur'10]
- Soundness proved by resorting to \vdash
- We discuss ongoing work on semantic completeness

Algorithmic type checking

- Well-known idea: in typing $P \mid Q$ remove linear identifiers used by P before type check Q (e.g. [Gay&Hole'05])
- Our approach: we reason at the type level and **forbid** (\circ) use of (parts of) types that have been:
 1. delegated
 2. consumed

$$(\sim ML) \quad \text{fun typeVar}(\Gamma, x : (\text{lin?}T_1.S_1, \text{lin!}T_2.S_2), x : \text{lin?}T_1.S_1) = \\ \Gamma, x : (\circ, \text{lin!}T_2.S_2)$$

ML patterns

- Typings for processes are patterns of function:

fun check(g : context, p : process) : context

- Patterns matching deterministic for *safe* types (no backtracking)
 1. dual unrestricted channel ends: $(*un?T, *un!T)$ and T safe
 2. dual linear ends: $(lin?T.S_1, lin!T.S_2)$ and T and (S_1, S_2) safe
- E.g.: typing an input process in linear mode

check($\Gamma, x : (lin?T.S_1, lin!T.S_2), x(y).P$) =
let val d = check($\Gamma, x : S_1, y : T, P$) in ...

Motivation of the design

- Algorithm works only for safe types = generalization *balancing*
- As usual, subject reduction only for balanced contexts
- The very reason is to preserve the soundness of the exchanges

$$P = x(y).\text{if } y \text{ then } \mathbf{0} \text{ else } \mathbf{0} \mid (\nu z : \text{end})\bar{x}\langle z \rangle.\mathbf{0}$$

$$x : (\text{lin?bool.end, lin!end.end}) \vdash P$$

$$P \rightarrow (\nu z : \text{end})\text{if } z \text{ then } \mathbf{0} \text{ else } \mathbf{0}$$

$$\not\vdash (\nu z : \text{end})\text{if } z \text{ then } \mathbf{0} \text{ else } \mathbf{0}$$

- Soundness: algorithm rejects non balanced types

Type checking algorithm

- The top level call accepts the process if:
 1. The environment in input is safe
 2. An environment is given in output (no patterns exception)
 3. The domain of the environment in output contains only *consumed* types of the form \circ , $\text{un } p$, $(\text{un } p_1, \text{un } p_2)$, $(\text{un } p, \circ)$, (\circ, \circ)

```
fun typeCheck( $\Gamma$  : context,  $P$  : process) : bool =  
  if safe( $\Gamma$ ) then  
    let val  $\Delta$  = check( $\Gamma$ ,  $P$ ) in  
      if consumed( $\Delta$ ) then true
```

A run

- Protocol described by concurrent execution of

$$Service = !poll(w).(\nu p : T) (\bar{w}\langle p \rangle.p(title).p(date).!p(date))$$
$$Invoker = \overline{poll}\langle y \rangle.y(p).(\bar{p}\langle Workshop \rangle.\bar{p}\langle 9June \rangle.(\bar{z}_1\langle p \rangle \mid \dots \mid \bar{z}_n\langle p \rangle))$$
$$S_1 = \text{lin} !\text{string}.\text{lin} !\text{date}. * \text{un} !\text{date}$$
$$S_2 = \text{lin} ?\text{string}.\text{lin} ?\text{date}. * \text{un} ?\text{date}$$
$$T = (S_1, S_2)$$
$$T_w = \text{lin} !S_1.\text{un} \text{end}$$

- Type checking succeeds

$$\text{typeCheck}(\Gamma, poll : (*\text{un} ?T_w, *\text{un} !T_w), Service \mid Invoker)$$

Checking the service continuation

- Replicated input spawns a thread for the poll

$$Service = !C$$
$$C = poll(w).(\nu p : T)\bar{w}\langle p \rangle.p(title).p(date).!p(date)$$

- Call requires environment in output = environment in input
- Intuition: only linear types change!

$$\begin{aligned} & \text{check}(\Gamma, Service) = \\ & \quad \text{let val } \Delta = \text{check}(\Gamma, C) \text{ in} \\ & \quad \text{if } (\Delta = \Gamma) \text{ then } \Delta \end{aligned}$$

Checking unrestricted input

- Service instantiation generates poll

$$C = \text{poll}(w).(\nu p : (S_1, S_2)) \bar{w}\langle p \rangle.p(\text{title}).p(\text{date}).!p(\text{date})$$

$$C' = (\nu p : (S_1, S_2)) \bar{w}\langle p \rangle.p(\text{title}).p(\text{date}).!p(\text{date})$$

$$T_w = \text{lin}!S_1.\text{un end}$$

- Call: type of channel does not change, bound variable added
- Return: checks types for the bound variable to be consumed

$$\text{check}(\Gamma, \text{poll} : (*\text{un} ?T_w, *\text{un} !T_w), C) =$$

$$\text{let val } \Delta = \text{check}(\Gamma, \text{poll} : (*\text{un} ?T_w, *\text{un} !T_w), w : T_w, C') \text{ in}$$

$$\text{if } (\Delta = \Delta', w : S) \text{ and } (S = \circ \text{ or } S = \text{un } p) \text{ then } \Delta'$$

Checking restriction

- A poll with safe channel type is generated

$$C' = (\nu p : (S_1, S_2)) \bar{w} \langle p \rangle . Q$$

$$\Omega = \Gamma, \text{poll} : (*\text{un } ?T_w, *\text{un } !T_w), w : T_w$$

- Call: add bound variable given that the type is safe
- Return: checks type for bound variable to be consumed

check(Ω , C') =

if **safe**((S_1, S_2)) then

let val $\Delta = \text{check}(\Omega, p : (S_1, S_2), \bar{w} \langle p \rangle . Q)$ in

if ($\Delta = \Delta', p : (S', S'')$)

and ($S' = \circ$ or $S' = \text{un } p$)

and ($S'' = \circ$ or $S'' = \text{un } p$)

then Δ'

Delegation of a linear session

- Poll write capability S_1 delegated over w of type $T_w = \text{lin } !S_1.\text{un end}$

$$\bar{w}\langle p \rangle.Q$$

- Call for the continuation
 1. session type for the channel unrolled
 2. delegated end point S_1 set to \circ by calling fun checkVar
- Return
 1. checks type for channel to be consumed
 2. returns context with type for channel set to \circ

$\text{check}(\Omega_1, w : T_w, p : (S_1, S_2), \bar{w}\langle p \rangle.Q) = \text{let... in}$

$\text{let val } \Delta = \text{check}(\Omega_1, w : \text{un end}, p : (\circ, S_2), Q) \text{ in}$

$\text{if } \Delta = \Delta', w : S \text{ and } S = \circ \text{ or } S = \text{un } p \text{ then } \Delta', w : \circ$

Checking parallel processes

- Concurrent delegation of poll channel to participants

$$P = \overline{z_1}\langle p \rangle \mid \overline{z_2}\langle p \rangle \mid \cdots \mid \overline{z_n}\langle p \rangle$$

$$S = *un!date$$

$$\Gamma = \Gamma_1, z_1 : lin!S.end, \cdots, z_n : lin!S.end, p : (\circ, S)$$

- Parallel processes typed compositionally

$$\text{check}(\Gamma, P) =$$

$$\text{let val } \Delta = \text{check}(\Gamma, \overline{z_1}\langle p \rangle) \text{ in}$$

$$\text{check}(\Delta, \overline{z_2}\langle p \rangle \mid \cdots \mid \overline{z_n}\langle p \rangle)$$

- In each call, the type of p in the input environment is (\circ, S)

Exchanging compositions' order

- Type checking the service protocol

$$\begin{aligned} \text{check}(\Gamma, \textit{Service} \mid \textit{Invoker}) = \\ \text{check}(\text{check}(\Gamma, \textit{Service}), \textit{Invoker}) \end{aligned}$$

- Preservation of structural congruence!

$$\text{check}(\Gamma, \textit{Invoker} \mid \textit{Service}) = \text{check}(\Gamma, \textit{Service} \mid \textit{Invoker})$$

Soundness

- We resort to declarative type system \vdash
- System \vdash relies on non deterministic operation to split contexts

$$\frac{\Gamma = \Gamma_1 \cdot \Gamma_2 \quad \Gamma_1, p : S_1 \vdash p : S_1 \quad \Gamma_2, w : \text{end}, p : S_2 \vdash Q}{\Gamma, p : (S_1, S_2), w : \text{lin} ! S_1.\text{end} \vdash \bar{w}\langle p \rangle.Q}$$

- $\text{typeCheck}(\Gamma, P)$ implies $\Gamma \vdash P$

Expressivity

- First, we type checked our motivating example
- Still, there are process accepted by \vdash that we do not type check

1. $\Gamma_1, x : (\text{lin } ?T.S_1, \text{lin } !T.S_2) \vdash \bar{x}\langle v \rangle.C[x(y).P]$

2. $\Gamma_2, x : (\text{lin } ?T.S_1, \text{lin } !T.S_2) \vdash x(y).C[\bar{x}\langle v \rangle.Q]$

3. $\Gamma_3, x : (\text{lin } ?T.S_1, \text{lin } !T.S_2) \vdash \bar{x}\langle x \rangle.P$

- But these processes are deadlocked!
- How to prove?

Typed behavioral theory

- In parallel work we proposed typed barbed equivalence for sessions

$$\Delta \models P \cong Q$$

1. $\Gamma_1 \vdash P, \Gamma_2 \vdash Q$
 2. Δ *compatible* with Γ_1, Γ_2 (e.g. no interference with a session)
 3. P and Q have same barbs in all contexts type checked by Δ
- Proof technique: typed bisimulation
 - Technical framework: polyadic pi calculus with matching, meet operation over types...

Application

- Let $\Gamma_i, x : (\text{lin } ?T.S_1, \text{lin } !T.S_2) \vdash P_i$ for $i = 1, 2, 3$
 - Let Δ_i be compatible with $\Gamma_i, x : (\text{lin } ?T.S_1, \text{lin } !T.S_2)$ for $i = 1, 2, 3$
1. $\Delta_1 \models \bar{x}\langle v \rangle.C[x(y).P] \cong \mathbf{0}$
 2. $\Delta_2 \models x(y).C[\bar{x}\langle v \rangle.Q] \cong \mathbf{0}$
 3. $\Delta_3 \models \bar{x}\langle x \rangle.P \cong \mathbf{0}$
- Wow! So, what?

Towards semantic completeness

- Proof transformation: $\Gamma_1 \vdash P_1$ transformed in $\Gamma_2 \vdash P_2$
- Construction: take derivation tree for $\Gamma_1 \vdash P_1$ and substitute each occurrence of $\Gamma, x : (\text{lin } ?T.S_1, \text{lin } !T.S_2) \vdash \bar{x}\langle v \rangle.Q$ with $\emptyset \vdash \mathbf{0}$
- Typed equivalence: Δ compatible with Γ_1, Γ_2 implies

$$\Delta \models P_1 \cong P_2$$

- Semantic completeness (in progress):

If $\Gamma_1 \vdash P_1$ with Γ_1 balanced, then there is a transformation $\Gamma_2 \vdash P_2$ such that $\Delta \models P_1 \cong P_2$ and $\text{typeCheck}(\Gamma_2, P_2)$.

Conclusions

- We introduced a type checking algorithm for the analysis of structured-oriented programs in the pi calculus
- Technique relies on construct that describes the two ends of the same channel
 - Each end point is a linear or an unrestricted session type
 - Linear types evolve to unrestricted types
- The algorithm is sound w.r.t. type system \vdash
- We claim to type check all interesting processes accepted by \vdash

Usefulness

- System \vdash enjoys type-preserving encodings of
 1. linear lambda calculus [Walker&05]
 2. linear pi calculus [KPT&TOPLAS'99]
 3. pi calculus with polarities [GH&Acta'05]
- We therefore offer an algorithm for typing functional and mobile languages based on linearity
- Other systems can be considered

$$\llbracket (\nu xy : S) P \rrbracket = (\nu x : (S, \overline{S})) \llbracket P[x/y] \rrbracket \quad [\text{V@SFM'09}]$$

Ongoing and future work

- Semantic completeness in progress
- Still, there are interesting processes that are not typable by \vdash
$$!x(y).(\nu a)(\bar{y}\langle a \rangle.a(\text{title}).a(\text{date}).(!a(\text{date}) \mid \bar{a}\langle 22\text{March} \rangle))$$
- Both capabilities needed in continuation for receive and send date
- Sub typing à la Pierce&Sangiorgi would fix this